

$$A^{50} = \begin{bmatrix} 2^{50} & -1+2^{50} & 0 \\ 0 & 1 & 0 \\ 1-2^{50} & 1-2^{50} & 1 \end{bmatrix}$$

Exercises:

Example 1.7.2.

$$\begin{bmatrix} 5 & -18 & -32 \\ 0 & 5 & 4 \\ +2 & -5 & -11 \end{bmatrix}$$

$$R_1 - 3R_3 \rightarrow R_1 : \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ $\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ I am doing this operation on the initial matrix

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & -18 & -32 \\ 0 & 5 & 4 \\ 2 & -5 & -11 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -3 & 1 \\ 0 & 5 & 4 \\ 2 & -5 & -11 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -3 & -2 \\ 0 & 5 & 4 \\ 2 & -5 & -5 \end{bmatrix}$$

B this matrix is similar to the initial matrix (they have same eigen values).

Now, we find the eigenvalues of B:

$$\det (XI - B) = 0$$

$$\det \left[X \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} -1 & -3 & -2 \\ 0 & 5 & 4 \\ 2 & -5 & -5 \end{pmatrix} \right] = 0$$

$$\det \begin{bmatrix} X+1 & 3 & -2 \\ 0 & X-5 & -4 \\ -2 & 5 & X+5 \end{bmatrix} = 0$$

~~Therefore,~~

$$(X+1)[(X-5)(X+5) - 20] - 2[-12 + (X-5)(2)] = 0$$

$$(X+1)[X^2 + 5X - 5X - 25 - 20] - 2[-12 + 2X - 10] = 0$$

$$(X+1)(X^2 - 45) - 2(-22 + 2X) = 0$$

$$X^3 - 45X + X^2 - 45 + 44 - 4X = 0$$

$$X^3 + X^2 - 49X - 1 = 0$$

$$(X+1)[X^2 - 5 + 4] = 0$$

$$(X+1)(X^2 - 1) = 0 \Rightarrow (X+1)(X-1)(X+1) = 0$$

$$X+1 = 0$$

$$X-1 = 0$$

$$X = -1$$

$$X = 1$$

$\lambda = -1$ mult. 2
 $\lambda = 1$ " 1

Eigenvalues of initial matrix A.

Now, we find the eigenvectors of A.

for $\lambda = -1$ $(\lambda I - A)X = 0$

$$\left[-1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 5 & -18 & -32 \\ 0 & 5 & 4 \\ 2 & -5 & -11 \end{pmatrix} \right] \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} -6 & 18 & 32 \\ 0 & -6 & -4 \\ -2 & 5 & 10 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

we transform it to an augmented matrix.

$$\left[\begin{array}{ccc|c} -6 & 18 & 32 & 0 \\ 0 & -6 & -4 & 0 \\ -2 & 5 & 10 & 0 \end{array} \right]$$

Now we should reduce it to the r.r.e.f.:

$$R_1 - 3R_3 \rightarrow R_3$$

$$\left[\begin{array}{ccc|c} -6 & 18 & 32 & 0 \\ 0 & -6 & -4 & 0 \\ 0 & 3 & 2 & 0 \end{array} \right]$$

$$R_2 + 2R_3 \rightarrow R_3$$

$$\left[\begin{array}{ccc|c} -6 & 18 & 32 & 0 \\ 0 & -6 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\Rightarrow R_1 + 3R_2 \rightarrow R_1 \left[\begin{array}{ccc|c} -6 & 0 & 20 & 0 \\ 0 & -6 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\frac{R_1}{-6} \rightarrow R_1 \left[\begin{array}{ccc|c} 1 & 0 & -\frac{10}{3} & 0 \\ 0 & -6 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\frac{R_2}{-6} \rightarrow R_2 \left[\begin{array}{ccc|c} 1 & 0 & -\frac{10}{3} & 0 \\ 0 & 1 & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \text{ r.r.e.f.}$$

$$E_{\lambda}(A) =$$

$$u_3 = t \quad (t \in \mathbb{R})$$

$$u_2 + \frac{2}{3} u_3 = 0$$

$$u_2 + \frac{2}{3} t = 0 \Rightarrow u_2 = -\frac{2}{3} t$$

$$u_1 - \frac{10}{3} u_3 = 0$$

$$u_1 - \frac{10}{3} t = 0 \Rightarrow u_1 = \frac{10}{3} t$$

$$X = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} \frac{10}{3} t \\ -\frac{2}{3} t \\ t \end{pmatrix} = t \begin{pmatrix} \frac{10}{3} \\ -\frac{2}{3} \\ 1 \end{pmatrix}_{3 \times 1}$$

eigenvector
corresponding to $\lambda = 1$

$$\dim(E_{-1}(A)) = +1 \leq \text{multiplicity of } \lambda = -1$$

So A is not diagonalisable. (we stop here)

Extra
for training

For $\lambda = 1$ what is the eigenvector

$$(\lambda I - A)X = 0$$

$$\left[\begin{array}{c|ccc} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] - \left[\begin{array}{ccc} 5 & -18 & -32 \\ 0 & 5 & 4 \\ 2 & -5 & -11 \end{array} \right] \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -4 & 18 & 32 \\ 0 & -4 & -4 \\ -2 & 5 & 2 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left[\begin{array}{ccc|c} -4 & 18 & 32 & 0 \\ 0 & -4 & -4 & 0 \\ -2 & 5 & 12 & 0 \end{array} \right]$$

$$R_3 - \frac{R_1}{2} \rightarrow R_3 \left[\begin{array}{ccc|c} -4 & 18 & 32 & 0 \\ 0 & -4 & -4 & 0 \\ 0 & -4 & -4 & 0 \end{array} \right]$$

$$R_3 - R_2 \rightarrow R_3 \left[\begin{array}{ccc|c} -4 & 18 & 32 & 0 \\ 0 & -4 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\frac{R_2}{-4} \rightarrow R_2 \left[\begin{array}{ccc|c} -4 & 18 & 32 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$R_1 - 18R_2 \rightarrow R_1 \left[\begin{array}{ccc|c} -4 & 0 & 14 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\frac{R_1}{-4} \rightarrow R_1 \left[\begin{array}{ccc|c} 1 & 0 & 7/2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$u_3 = t \quad (t \in \mathbb{R})$$

$$u_2 + u_3 = 0$$

$$u_2 = -u_3$$

$$\Rightarrow u_2 = -t$$

$$u_1 - \frac{7}{2}t = 0$$

$$u_1 = \frac{7}{2}t$$

$$X = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 7/2 t \\ -t \\ t \end{pmatrix} = t \begin{pmatrix} 7/2 \\ -1 \\ 1 \end{pmatrix} \quad \text{eigenvector of } \lambda = 1$$

Exercise 7.2.3.

$$A = \begin{bmatrix} 89 & 38 & 268 \\ 14 & 2 & 40 \\ -30 & -12 & -90 \end{bmatrix}$$

$$R_1 + 3R_3 \rightarrow R_1: \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P^{-1}AP^{-1} = D$$

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 89 & 38 & 268 \\ 14 & 2 & 40 \\ -30 & -12 & -90 \end{pmatrix} \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 2 & -2 \\ 14 & 2 & 40 \\ -30 & -12 & -90 \end{pmatrix} \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 2 & 1 \\ 14 & 2 & -2 \\ -30 & -12 & 0 \end{pmatrix}$$

Now, we find the eigenvalues of B:

$$\det (xI - A) = 0$$

det $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Ex 3.1)

Remark: If we choose $P = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow P^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$

$$D = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix} \quad = \begin{bmatrix} 1/2 & -1/2 \\ -1/2 & -1/2 \end{bmatrix}$$

(yinkin y zelna 1 eigenvectors w yinkin
y zelna nbanhen ano zet kshi)

Now si on a change of order, A^{*m} will be the same

$$A^{*0} = P D^{*0} P^{-1}$$

Suite Exercise 7.2.3 :

$$\det \left[X \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 2 & 1 \\ 14 & 2 & -2 \\ -30 & -12 & 0 \end{pmatrix} \right] = 0$$

$$\begin{bmatrix} X+1 & -2 & -1 \\ -14 & X-2 & 2 \\ 30 & 12 & X \end{bmatrix} = 0$$

$$\begin{array}{r} 14 \\ \times 12 \\ \hline 28 \\ + 140 \\ \hline 168 \end{array}$$

$$(X+1)[X(X-2)+24] + 2[-14X+60] - 1[(-14 \times 12) + 30(X-2)] = 0$$

$$(X+1)(X^2 - 2X + 24) - 28X + 120 - 1(-168 + 30X - 60) = 0$$

$$(X+1)(X^2 - 2X + 24) - 28X + 120 + 168 - 30X + 60 = 0$$

$$(X+1)(X^2 - 2X + 24) - 58X + 348 = 0$$

$$\begin{array}{r} 560 \\ - 24 \\ \hline 36 \\ + 348 \\ \hline 372 \end{array}$$

$$X^3 - 2X^2 + 24X + X^2 - 2X + 24 - 58X + 348 = 0$$

$$X^3 - X^2 - 36X + 372 = 0$$

$$X(X^2 - X - 36) + 372 = 0$$

Exercice 7.2.2:

à trouver?

$$A = \begin{bmatrix} -13 & -28 & 28 \\ 4 & 9 & -8 \\ -4 & -8 & 9 \end{bmatrix}$$

① ~~car~~ $\det(XI - A) = 0$

$$\det \left(X \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -13 & -28 & 28 \\ 4 & 9 & -8 \\ -4 & -8 & 9 \end{bmatrix} \right) = 0$$

$$\det \begin{bmatrix} X+13 & 28 & -28 \\ -4 & X-9 & 8 \\ 4 & 8 & X-9 \end{bmatrix} = 0$$

$$R_2 + R_3 \rightarrow R_3 \quad \begin{bmatrix} X+13 & 28 & -28 \\ -4 & X-9 & 8 \\ 0 & X-1 & X-1 \end{bmatrix} = 0$$

$X-9+8=X$

$$(X+13) \left[\cancel{28(X-1)} (X-9)(X-1) - 8(X-1) \right] - (-4) \left[28(X-1) - 28(X-1) \right] = 0$$

$$(X+13) \left[X^2 - X - 9X + 9 - 8X + 8 \right] + 4(0) = 0$$

$$(X+13)(X^2 - 17X + 17) = 0$$

$$\textcircled{0} (X+13)(X^2 - 17(X+1)) = 0$$

$$X+13=0$$

$$X = -13$$

$$X^2 = 0$$

IFE

$$-17(X+1) = 0$$

$$\textcircled{0} X+1=0$$

$$X = -1$$

E. 7.3. 2: is an upper triangular matrix, so the values are the entries of the diagonal: $\lambda_1=1, \lambda_2=2, \lambda_3=5$.

$\lambda = 0, \lambda = -13, \lambda = -1$ are the eigenvalues of A.

are the eigenvalues

for $\lambda=0$

$$(\lambda I - A)X = 0$$

$$\left[0 I - \begin{pmatrix} -13 & -28 & 28 \\ 4 & 9 & -8 \\ -4 & -8 & 9 \end{pmatrix} \right] \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$\left[\begin{array}{ccc|c} 13 & 28 & -28 & 0 \\ -4 & -9 & 8 & 0 \\ 4 & 8 & 9 & 0 \end{array} \right]$$

$$R_2 + R_3 \rightarrow R_2 \quad \left[\begin{array}{ccc|c} 13 & 28 & -28 & 0 \\ 0 & -1 & 17 & 0 \\ 0 & 8 & 9 & 0 \end{array} \right]$$

~~$R_1 \leftrightarrow R_3$~~
 ~~$R_1 \leftrightarrow R_3$~~
 ~~$R_1 \leftrightarrow R_3$~~

$R_1 \leftrightarrow R_3 \rightarrow R_3$

$$\begin{array}{r} 27 + 28 \\ 27 \\ \hline 55 \end{array}$$

$$\left[\begin{array}{ccc|c} 13 & 28 & -28 & 0 \\ 0 & -1 & 17 & 0 \\ +1 & +4 & -55 & 0 \end{array} \right]$$

$R_3 \leftrightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 4 & -55 & 0 \\ 0 & -1 & 17 & 0 \\ 13 & 28 & -28 & 0 \end{array} \right]$$

$13R_1 - R_3 \rightarrow R_3$

$$\left[\begin{array}{ccc|c} 1 & 4 & -55 & 0 \\ 0 & -1 & 17 & 0 \\ 0 & 24 & 687 & 0 \end{array} \right]$$

$$\begin{array}{r} 13 \times 55 \\ 715 \\ 24 \\ \hline 687 \end{array}$$

$$ax^2 + bx + c \quad \lambda = -3 \Rightarrow a(n-u_1)(n-u_2)$$

Exercise 7.2.3:

$$A = \begin{bmatrix} 89 & 38 & 268 \\ 14 & 2 & 40 \\ -30 & -12 & -90 \end{bmatrix}$$

$$3R_3 + R_1 \rightarrow R_1: \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P^{-1}AP =$$

$$\begin{array}{r} 12 \quad 90 \\ 3 \quad 3 \\ \hline 36 \quad 270 \end{array}$$

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 89 & 38 & 268 \\ 14 & 2 & 40 \\ -30 & -12 & -90 \end{pmatrix} \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 2 & -2 \\ 14 & 2 & 40 \\ -30 & -12 & -90 \end{pmatrix} \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{array}{r} 14 \\ 3 \\ \hline 42 \\ 40 \end{array}$$

$$= \begin{pmatrix} -1 & 2 & 1 \\ 14 & 2 & -2 \\ -30 & -12 & 0 \end{pmatrix}$$

$$2R_2 + R_3 \rightarrow R_3: \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} -1 & 2 & 1 \\ 14 & 2 & -2 \\ -30 & -12 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 2 & 1 \\ 14 & 2 & -2 \\ -2 & -8 & -4 \end{pmatrix}$$

1 - 7.3.1 - 7.3.2

$$\det (XI - A) = 0$$

$$\det \left[X \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 2 & 1 \\ 14 & 2 & -2 \\ -2 & -8 & -4 \end{pmatrix} \right] = 0$$

$$\det \begin{bmatrix} X+1 & -2 & -1 \\ -14 & X-2 & 2 \\ 2 & 8 & X+4 \end{bmatrix} = 0$$

$$4R_1 + R_3 \rightarrow R_1:$$

$$\det \begin{bmatrix} 4X+6 & 0 & X \\ -14 & X-2 & 2 \\ 2 & 8 & X+4 \end{bmatrix} = 0$$

$$4X+4+2 = 4X+6$$

$$-8+8 = 0$$

$$4+X+4$$

$$\frac{14}{2} = 7$$

$$(4X+6) \left[(X-2)(X+4) - 16 \right] + X \left[-112 - 2(X-2) \right] = 0$$

$$(4X+6) (X^2+4X-2X-12-16) - 112X - 2X^2 + 4X = 0$$

$$(4X+6) (X^2+2X-28) - 108X - 2X^2 = 0$$

$$4X^3 + 8X^2 - 112X + 6X^2 + 12X - 168 - 108X - 2X^2 = 0$$

$$4X^3 + 12X^2 - 110X - 276 = 0$$

$$\frac{28}{2} = 14$$

$$\frac{28}{2} = 14$$